An Application of the Individual Channel Analysis and Design Approach to Control of a Two-input Two-output Coupled-tanks System

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Abstract

Frequency-domain methods have provided an established approach to the analysis and design of single-loop feedback control systems in many application areas for many years. Individual Channel Analysis and Design (ICAD) is a more recent development that allows neo-classical frequency-domain analysis and design methods to be applied to multi-input multi-output control problems. This paper provides a case study illustrating the use of the ICAD methodology for an application involving liquid-level control for a system based on two coupled tanks. The complete nonlinear dynamic model of the plant is presented for a case involving two input flows of liquid and two output variables, which are the depths of liquid in the two tanks. Linear continuous proportional plus integral controllers are designed on the basis of linearised plant models to meet a given set of performance specifications for this two-input two-output multivariable control system and a computer simulation of the nonlinear model and the controllers is then used to demonstrate that the overall closed-loop performance meets the given requirements. The resulting system has been implemented in hardware and the paper includes experimental results which demonstrate good agreement with simulation predictions. The performance is satisfactory in terms of steady-state behaviour, transient responses, interaction between the controlled variables, disturbance rejection and robustness to changes within the plant. Further simulation results, some of which involve investigations that could not be carried out in a readily repeatable fashion by experimental testing, give support to the conclusion that this neo-classical ICAD framework can provide additional insight within the analysis and design processes for multi-input multi-output feedback control systems.

Keywords: multivariable, feedback, control, coupled tanks, frequency domain, nonlinear.

1 Introduction

Problems of liquid level control arise in many industries. Common examples include control of levels in blending and reaction vessels within chemical processes. This paper describes the application of the Individual Channel Analysis and Design (ICAD) approach to the development, implementation and testing of conventional diagonal controllers for a system involving liquid levels in a pair of coupled tanks [1]. The objective of the paper is to provide a detailed case-study to illustrate use of the ICAD methodology and to demonstrate some of the benefits of this neo-classical frequency-domain approach to problems involving multivariable control.

The coupled-tanks equipment around which the control system is designed has two inputs, which are external liquid flow-rates into each of the tanks. The two outputs are the resulting levels of liquid in the tanks. There is only one outflow and this is from the second tank. Figure 1 is a schematic diagram of this system.

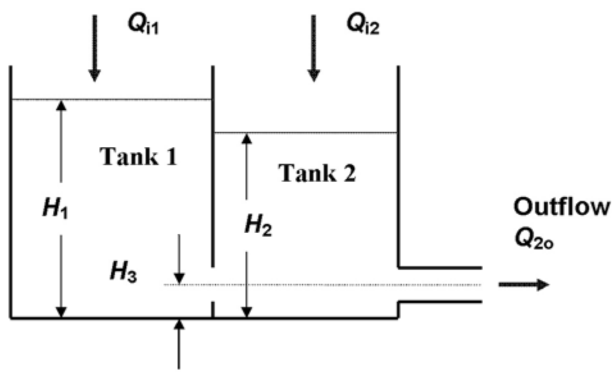


Figure 1: Schematic diagram of the coupled-tanks equipment

Established frequency-domain methods, such as Bode/Nyquist analysis, are of central importance as a basis for design tools for single-input single-output feedback control systems in many application areas. The success of the frequency-domain approach is due, in part, to the graphical nature of these techniques which provides transparency and flexibility in satisfying design specifications in the presence of practical constraints. The extension of classical methods of analysis and design to multivariable systems involving more than one input and more than one output can introduce difficulties. It may still be possible, in cases where cross-coupling is not strong, to design the control system using approaches involving one loop at a time. However, in cases where dynamic interactions between loops are significant, more skill and experience is necessary in order to produce a successful design, and a process of tuning using trial-and-error methods may be needed.

The Individual Channel Analysis and Design (ICAD) methodology was developed in the early 1990s and allows frequency-domain methods to be applied to the problems of analysis and design of multi-input multi-output feedback control systems (see, e.g., [2–6]). This approach allows an m-input m-output feedback control problem to be split into m single-input single-output problems without loss of structural information. Each controlled output is paired with a specific reference input to form what is termed a "Channel". The ICAD approach makes direct use of the customer performance specification for different channels to provide a framework within which classical single-input single-output control engineering concepts can be extended to multi-input multi-output cases involving significant levels of cross-coupling. Traditional methods for applications involving single-input single-output (SISO) systems can thus be applied to multi-input multi-output (MIMO) problems. This includes the use of open-loop system information in the form of Nyquist and Bode plots for system analysis and design, along with simple measures of robustness, such as gain and phase margins.

It must be emphasized that the ICAD approach is not, itself, a design method but should be viewed as a framework through which useful insight may be gained about the dynamics of the plant and the characteristics of the complete controlled system. Performance can be assessed for any chosen form of linear controller (which may be designed using any suitable approach), and limitations of a design can be investigated. Thus, compared with some other approaches to multivariable control, it can be based on traditional analysis and design methods familiar to all control system designers. Each channel has its own customer-defined performance specifications and these may be expressed in a simple way in terms of SISO requirements.

2 The coupled-tanks system

The two-input coupled-tanks laboratory system of Figure 1 consists of a container (of volume 6 liters), with a central partition which divides it into two separate tanks. Coupling between these tanks is provided by a number of holes of various diameters positioned near the base of the partition. The strength of coupling may be adjusted through the insertion of plugs into one or more of these holes. The system is equipped with a drain tap which is under manual control and this allows the output flow rate from one of the tanks to be adjusted. Both tanks have inflows from electrically driven variable-speed pumps and are equipped with sensors that can detect the level of liquid and provide a proportional electrical output voltage.

The hardware is based around a single-input commercial product intended for teaching applications (TecQuipment Ltd) [1]. This had a flow input only to Tank 1 and was modified at the University of Glasgow through the addition of the second pump to provide the inflow to Tank 2. The original resistive level sensors have been replaced using more accurate differential-pressure based depth sensors.

The derivation of a detailed nonlinear model of the system may be found in [7] and in a more recent conference paper [8] which also includes a very brief account of the application of the ICAD approach to the design of a control system for this process.

The model is based on the application of the principle of conservation of mass to the liquid within each tank. Bernoulli's equations provide the basis for determining the flow from one tank to the other and from the second tank to the external environment. This leads to the following pair of equations, involving variables shown in Figure 1:

$$A_1 \frac{\mathrm{d}H_1}{\mathrm{d}t} = Q_{i1} - C_{d1} a_1 \sqrt{2g(H_1 - H_2)} \tag{1}$$

$$A_2 \frac{\mathrm{d}H_2}{\mathrm{d}t} = Q_{i2} + C_{d1} a_1 \sqrt{2g(H_1 - H_2)} - C_{d2} a_2 \sqrt{2g(H_2 - H_3)} \tag{2}$$

These equations describe the dynamics of the coupled tanks system, in nonlinear form, for all cases for which the level in Tank 2 is below that in Tank 1. It is, of course, possible to derive a similar set of nonlinear equations to describe the system for cases involving a liquid level in Tank 2 which is greater than the level in Tank 1.

Parameter values for the laboratory system are as follows:

Cross-sectional area of tanks, $A_1 = A_2 = 9.7 \times 10^{-3}\ \mathrm{m}^2$.

Cross-sectional area of the orifice between Tank 1 and Tank 2, $a_1 = 3.956 \times 10^{-5}\ \mathrm{m}^2$.

Cross-sectional area of the orifice representing the outlet drain of Tank 2, $a_2 = 3.85 \times 10^{-5}\ \mathrm{m}^2$.

Height of the outlet drain above the base of Tank 2, $H_3 = 0.03$ m.

Gravitational constant, $g = 9.81\ \mathrm{ms}^{-2}$.

Maximum flow rates for inputs to Tank 1 and Tank 2:

$Q_{i1\,\max} = Q_{i2\,\max} = 5 \times 10^{-5}\ \mathrm{m}^3\ \mathrm{s}^{-1}$.

The minimum flow rates for these inputs are zero since the pumps are not reversible and thus negative inputs are not possible.

Maximum levels of liquid in Tank 1 and Tank 2: $H_{1\,\max} = H_{2\,\max} = 0.3$ m.

The minimum level possible in each tank is 0.03 m which corresponds to the height of the outlet drain.

Values of the discharge coefficients C_{d1} and C_{d2} have to be determined empirically and the values obtained depend on the system operating point. The value used for C_{d1} in the design calculations was 0.63 and the value used for C_{d2} was 0.58.

In addition to the above parameters, electrical signals in the system are related to the variables of the model (as described by Equations (1) and (2)) through the following two parameters:

Pump flow-rate calibration constant, $G_p = 7.2 \times 10^{-6}\ \mathrm{m^3 s^{-1} V^{-1}}$.

Liquid depth sensor calibration constant, $G_d = 33.33\ \mathrm{Vm^{-1}}$.

It should be noted that no dynamic representation is included for the two pumps and the associated electrical drives as it was found, through hardware testing, that they show a very fast response to changes of electrical input compared with the level changes within the tanks themselves. Time constants that are associated with the pumps were therefore neglected for the purposes of the control system design.

For the preliminary stages of the design it is also appropriate to consider a linearised model, within which the variables represent small variations of system variables about steady state values.

$$\tilde{h}_1(t) = \bar{H}_1 - H_1(t) \tag{3}$$
$$\tilde{h}_2(t) = \bar{H}_2 - H_2(t) \tag{4}$$
$$q_{i1}(t) = \bar{Q}_{i1} - Q_{i1}(t) \tag{5}$$
$$q_{i2}(t) = \bar{Q}_{i2} - Q_{i2}(t) \tag{6}$$
$$q_{23}(t) = \bar{Q}_{23} - Q_{23}(t) \tag{7}$$

In Equations (3)–(7) the variables that have a horizontal bar above them denote values at the chosen operating point, which is normally defined by a steady-state condition.

If Equations (1) and (2) are re-arranged in the standard state-space form, we get a pair of nonlinear equations:

$$\frac{\mathrm{d}H_1}{\mathrm{d}t} = f_1(H_1, H_2, Q_{i1}) \tag{8}$$
$$\frac{\mathrm{d}H_2}{\mathrm{d}t} = f_2(H_1, H_2, H_3, Q_{i2}) \tag{9}$$

Then, since the level H_3 may be assumed constant, linearisation produces the standard linear state space model:

$$\begin{bmatrix} \dfrac{d\tilde{h}_1}{\mathrm{d}t} \\ \dfrac{d\tilde{h}_2}{\mathrm{d}t} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1}{\partial H_1} & \dfrac{\partial f_1}{\partial H_2} \\ \dfrac{\partial f_2}{\partial H_1} & \dfrac{\partial f_2}{\partial H_2} \end{bmatrix} \begin{bmatrix} \tilde{h}_1 \\ \tilde{h}_2 \end{bmatrix} + \begin{bmatrix} \dfrac{\partial f_1}{\partial Q_1} & \dfrac{\partial f_1}{\partial Q_2} \\ \dfrac{\partial f_2}{\partial Q_1} & \dfrac{\partial f_2}{\partial Q_2} \end{bmatrix} \begin{bmatrix} q_{i1} \\ q_{i2} \end{bmatrix} \tag{10}$$

In Equation (10) all the partial derivatives must be evaluated at the operating point $(\bar{H}_1, \bar{H}_2, \bar{Q}_{i1}, \bar{Q}_{i2})$.

The resulting linearised equation, after evaluation of the partial derivatives, has the form:

$$\begin{bmatrix} \dot{\tilde{h}}_1 \\ \dot{\tilde{h}}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{-\alpha_1}{A_1} & \dfrac{\alpha_1}{A_1} \\ \dfrac{\alpha_1}{A_2} & \dfrac{-(\alpha_1+\alpha_2)}{A_2} \end{bmatrix} \begin{bmatrix} \tilde{h}_1 \\ \tilde{h}_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{A_1} & 0 \\ 0 & \dfrac{1}{A_2} \end{bmatrix} \begin{bmatrix} q_{i1} \\ q_{i2} \end{bmatrix} \tag{11}$$

where

$$\alpha_1 = \frac{C_{d1} a_1}{2} \sqrt{\frac{2g}{\bar{H}_1 - \bar{H}_2}} \tag{12}$$

and

$$\alpha_2 = \frac{C_{d2} a_2}{2} \sqrt{\frac{2g}{\bar{H}_2 - H_3}} \tag{13}$$

The individual block transfer functions that describe the plant in Figure 1 may then be derived from Equation (11) and are as follows:

$$g_{11}(s) = \frac{\frac{(\alpha_1+\alpha_2)}{\alpha_1\alpha_2}\left[1 + s\frac{A_2}{(\alpha_1+\alpha_2)}\right]}{1 + \frac{(A_1\alpha_1 + A_1\alpha_2 + A_2\alpha_1)}{\alpha_1\alpha_2}s + \frac{A_1A_2}{\alpha_1\alpha_2}s^2} \tag{14}$$

$$g_{21}(s) = \frac{\frac{1}{\alpha_1}}{1 + \frac{(A_1\alpha_1 + A_1\alpha_2 + A_2\alpha_1)}{\alpha_1\alpha_2}s + \frac{A_1A_2}{\alpha_1\alpha_2}s^2} \tag{15}$$

$$g_{12}(s) = \frac{\frac{1}{\alpha_2}}{1 + \frac{(A_1\alpha_1 + A_1\alpha_2 + A_2\alpha_1)}{\alpha_1\alpha_2}s + \frac{A_1A_2}{\alpha_1\alpha_2}s^2} \tag{16}$$

$$g_{22}(s) = \frac{\frac{1}{\alpha_2}(1 + s\frac{A_1}{\alpha_1})}{1 + \frac{(A_1\alpha_1 + A_1\alpha_2 + A_2\alpha_1)}{\alpha_1\alpha_2}s + \frac{A_1A_2}{\alpha_1\alpha_2}s^2} \tag{17}$$

3 An outline of the ICAD approach

A linear time-invariant MIMO plant may be modelled using a transfer function matrix G. If a control matrix K is positioned in the forward path in cascade with the plant transfer function matrix G and immediately before it, a feedback loop can be created around the combined system described by the product KG. The essential feature of the ICAD approach is that loops are considered individually, by opening one loop while all other loops remain closed.

Details of the ICAD methodology and applications that have been considered previously may be found in papers by Leithead and O'Reilly (e.g. [2–4] and [5]), who were responsible for the initial development of the approach. A bibliography of published papers and reports relating to ICAD methods has been made available by Kocijan [6].

The ICAD methodology allows a controller to be assessed in a very direct fashion, for a given plant and given design specifications, in terms of performance and in terms of compromises and possible trade-offs. The design goals typically may involve:

1. Steady state response
2. Transient response
3. Disturbance rejection
4. Closed-loop stability
5. Robustness to changes in plant characteristics
6. Protection of actuators from high-frequency signals that might lead to excessive wear

In the ICAD approach the significance of the 'structure' of the plant in translating the given MIMO system into the equivalent set of channels is given special emphasis [2].

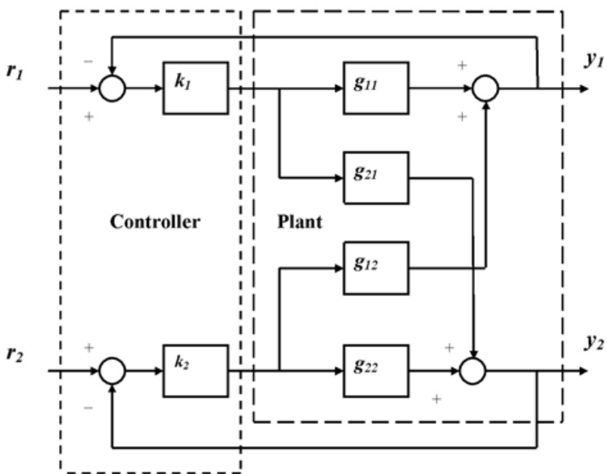


Figure 2: Block diagram of a general two-input two-output closed-loop system of the type being considered in this application. (Adapted from a diagram in [2])

As is clear from the plant model, the coupled-tanks application described in this paper involves a two-input two-output system with feedback involving two channels. Figure 2 is a block diagram that illustrates the type of system being investigated. If we consider the forward signal transmission from the reference signal r_1 to the associated output y_1, it may be seen that the signal follows two pathways. One path involves a direct link through the block g_{11} and the other is through the blocks g_{21}, g_{12} and a block involving k_2 and its associated feedback loop through g_{22}. This diagram may be simplified to give the structure shown in Figure 3 for the Channel C_1. From considerations of symmetry, the Channel C_2 may be handled in the same way to produce the simplified block diagram of Figure 4.

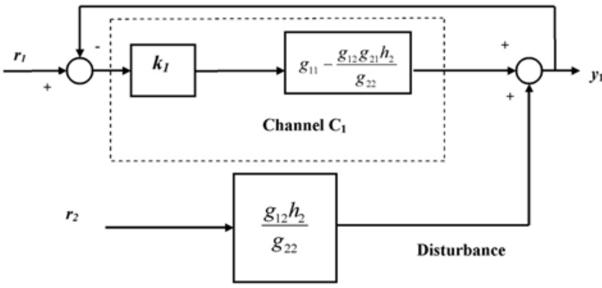


Figure 3: Block diagram for Channel 1. (Adapted from a diagram in [2])

These block diagrams can be used to show that, ignoring the disturbance signal, each channel can be described using a single-input single-output transfer function:

$$C_1 = k_1 g_{11}(1 - \gamma h_2) \tag{18}$$

and

$$C_2 = k_2 g_{22}(1 - \gamma h_1) \tag{19}$$

where

$$\gamma = \frac{g_{12} g_{21}}{g_{11} g_{22}} \tag{20}$$

$$h_2 = \frac{k_2 g_{22}}{1 + k_2 g_{22}} \tag{21}$$

and

$$h_1 = \frac{k_1 g_{11}}{1 + k_1 g_{11}} \tag{22}$$

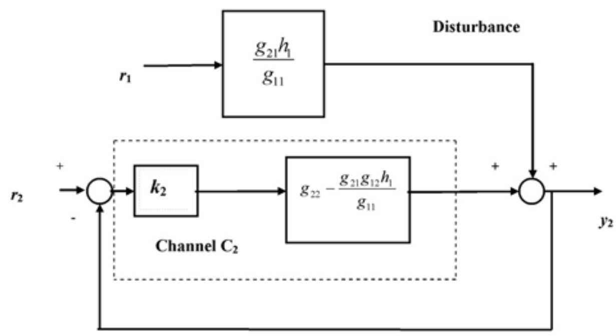


Figure 4: Block diagram for Channel 2. (Adapted from a diagram in [2])

In Equations (18)–(22) and in Figures 3 and 4, the effects of coupling are represented as additive disturbance terms at the outputs of each channel and this does not involve any loss of information. However, it must be emphasised that ICAD is not a single-loop design method since loop interactions are preserved.

It can be shown (e.g., [2, 4]) that, for robustness to parameter uncertainties of the closed-loop system stability, the Nyquist plots of $(1-\gamma h_1)$ and $(1-\gamma h_2)$ must not lie close to the origin of the polar plane at frequencies near to or below the open-loop gain cross-over frequency. Hence, if the corresponding plots of $\gamma h_1(j\omega)$ or $\gamma h_2(j\omega)$ come close to the point $(1, 0)$ in the polar plane the conventional SISO gain margins for the effective transfer functions of C_1 and C_2 do not provide robust measures of stability. In such a case it may not be appropriate to attempt to use the ICAD approach unless some form of pre-compensator is introduced to modify the plant characteristics in an appropriate way [5].

It can also be stated [4] that the quantities $h_1(j\omega)$ and $h_2(j\omega)$ have magnitude values that are generally close to one below the gain crossover frequency, and the quantity $\gamma(j\omega)$, which is termed the *multivariable structure function*, provides a measure of the strength of any inter-loop coupling in the system and can indicate whether or not this is benign. For the case of a two-input two-output system there is only one multivariable structure function. However, in general, for systems having a larger number of input-output pairs there will be more than one structure function.

In all cases it can be stated that when a multivariable structure function is small the interaction effects are small. In the two-input two-output case, if the multivariable structure function is small over the complete frequency range of interest, the two channels behave, more or less, as two independent loops. On the other hand, if the multivariable structure function is shown to have a large magnitude, at a frequency within the range that is important for the application being considered, the loop interactions become significant [2].

If the multivariable structure function has an appropriate form, as discussed above, frequency response information for each channel can be used in the analysis of the nominal system in exactly the same way as for the analysis of a conventional feedback loop in a SISO control system application. However, the multivariable structure function provides additional information about potential interactions and the stability robustness of the closed-loop system.

It is important to note that, for successful application of the ICAD approach to a two-channel system, the closed-loop bandwidth specification for one channel must not be too similar to the equivalent specification for the second channel. If this is not the case the problem of the transfer function of one channel depending on the controller of the other channel becomes a significant obstacle in the processes of analysis and design [2].

4 Design of the controller for the coupled-tanks system using ICAD

For the coupled-tanks system, it may be shown that the multivariable structure function is given by:

$$\gamma(s) = \frac{g_{12}g_{21}}{g_{11}g_{22}} = \frac{\frac{\alpha_2}{\alpha_1+\alpha_2}}{(1+s\frac{A_1}{\alpha_1})(1+s\frac{A_2}{\alpha_1+\alpha_2})} \tag{23}$$

The expressions for $h_1(s)$ and $h_2(s)$ for this system may also be derived directly, from Equations (21) and (22).

4.1 Design requirements

The specifications for the closed-loop system were based on equivalent requirements for a SISO version of the coupled-tanks system, for which considerable previous design experience had been accumulated. The requirements for the two-input two-output case were as follows:

a) Zero steady-state errors in the liquid levels in both tanks.
b) A maximum overshoot of 30 % in liquid levels.
c) A damping factor of at least 0.7 which corresponds, approximately, to a phase margin of at least 70 degrees for both channels.
d) The gain cross-over frequency should be at least 0.05 rad/s for both channels. This value is based on previous experience with the design of PID controllers for the SISO case for control of the liquid level in Tank 2 (with controlled input flow to Tank 1 only),
e) For successful application of the ICAD design methodology, it is important to ensure that the polar plots of the multivariable structure functions $\gamma(j\omega)$, $\gamma h_1(j\omega)$ and $\gamma h_2(j\omega)$ (in terms of the magnitude and phase values at different frequencies over the frequency range of significance) never approach the point $(1, 0)$.

4.2 An outline of the design process

The requirements outlined above provide a basis for design using the ICAD methodology for the linearised plant model, for selected operating conditions. Design, in this case, has involved the use of Matlab® software and has led to continuous and digital controllers involving proportional plus integral controller structures for each channel.

The design process was carried out for parameter values of the linearised model which correspond to an operating point in the lower half of the depth range in both tanks ($H_1 = 0.115$ m and $H_2 = 0.071$ m). This is a typical operating point for the system under open-loop conditions. The values used for the two discharge coefficients are those given in Section 2.

The first step in the design process involves establishing that the gain cross-over frequency of the open-loop transmittance of one channel will be significantly different from the gain cross-over frequency of the other. In this case it was decided, on the basis of physical reasoning, that the gain cross-over frequency of Channel 1 should be higher than that for Channel 2. From the design requirements, this latter value should be chosen to be at least 0.05 rad/s, so a value of at least 0.5 rad/s was required for the gain crossover frequency of Channel 1.

The next step involves evaluation of the magnitude and phase of the multivariable structure function over the range of frequencies that are of importance for the intended application. Figure 5 is a typical plot of the multivariable structure function in polar form showing the magnitude and phase of $\gamma(j\omega)$ for the complete range of relevant frequencies, and it is clear that the resulting plot involves small values of magnitude and does not come close to the $(1, 0)$ point. This is satisfactory for the operating point considered but similar plots should be considered for a range of different operating conditions.

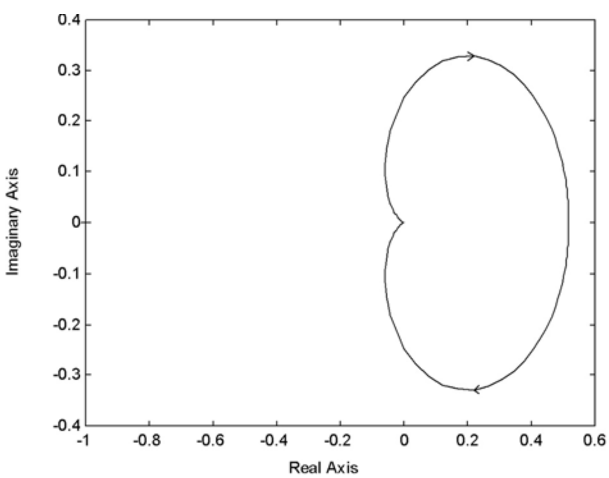


Figure 5: Plot of the multivariable structure function in polar form for the coupled-tanks system showing the magnitude and phase of $\gamma(j\omega)$ for the complete range of relevant frequencies for one operating point

Next, it is necessary to design the controller $k_2(s)$ since the requirements in terms of gain cross-over frequency for Channel 2 are less demanding than for Channel 1. Equation (19) shows that the equation for Channel 2 involves the transfer function $h_1(s)$ and the known multivariable structure function $\gamma(s)$. The first step is to assume either that $h_1(s) = 0$ or that $h_1(s) = 1$ and design the controller $k_2(s)$ initially on that basis [2]. In this application it was assumed that $h_1(s) = 0$, but an initial assumption that $h_1(s) = 1$ would have been equally appropriate. The transfer function for $g_{22}(s)$ given in Equation (17) has a magnitude at low frequencies of $1/\alpha_2$ and at high frequencies the magnitude decreases in an approximately linear fashion at -20 dB per decade. In order to meet the design requirement of zero steady-state closed-loop error this suggests use of a proportional plus integral type of controller of the form:

$$\beta_1 = \frac{(1 + \beta_2 s)}{s} \tag{24}$$

where β_1 and β_2 are constants.

This controller will produce infinite gain at zero frequency and thus eliminate any steady state error in the closed-loop system for this channel. The choice of parameter values for the controller involves, initially, the selection of a gain factor β_1 to give a suitable gain cross-over frequency which is at least the required minimum of 0.05 rad/s. The integral action is then considered and the frequency $\omega = \dfrac{1}{\beta_2}$ is chosen to be sufficiently smaller than the gain cross-over frequency to ensure that the overall phase margin is not influenced to any large extent. Application of this procedure gives the following controller:

$$k_2(s) = 0.56\frac{(1 + 8.929s)}{s} \tag{25}$$

Through the use of simulation, closed-loop step responses can be examined (usually on the basis of linearised models) and further adjustments can be made in the values for these controller parameters if this is judged to be necessary.

After obtaining that first approximation to $k_2(s)$ an initial single-input single-output design can be carried out for the controller $k_1(s)$ on the basis of $h_2(s)$, which is now available (from Equation (21)). The procedure followed is essentially the same as for Channel 1 but with the higher value of gain crossover frequency that is required for this channel. The proportional plus integral controller resulting initially from this process has the form:

$$k_1(s) = 4.676\frac{(1 + 5.988s)}{s} \tag{26}$$

Having found an initial $k_1(s)$, this transfer function can then be used to determine $h_1(s)$ by substitution into Equation (22). The resulting gain and phase margins must then be checked and adjustments made to $k_1(s)$, if necessary. The process may have to be repeated once or twice. Then, using the revised controller transfer function for Channel 1, the design can be completed for Channel 2 using a similar iterative procedure. Final checks must then be made on both channels to compare the gain cross-over frequencies with the design specifications and check that the gain and phase margins are all satisfactory. This process also involves re-examination of the Nyquist plots of the multivariable structure functions $\gamma(j\omega)$, $\gamma h_1(j\omega)$ and $\gamma h_2(j\omega)$ to ensure that none of them approaches the point $(1, 0)$ and thus establish that the gain and phase margins are valid measures of robustness [2].

Following the application of the above procedures the optimised controller transfer functions were as follows:

$$k_1(s) = 5.0\frac{(1+6.2s)}{s} \quad (27)$$

$$k_2(s) = 0.56\frac{(1+10.0s)}{s} \quad (28)$$

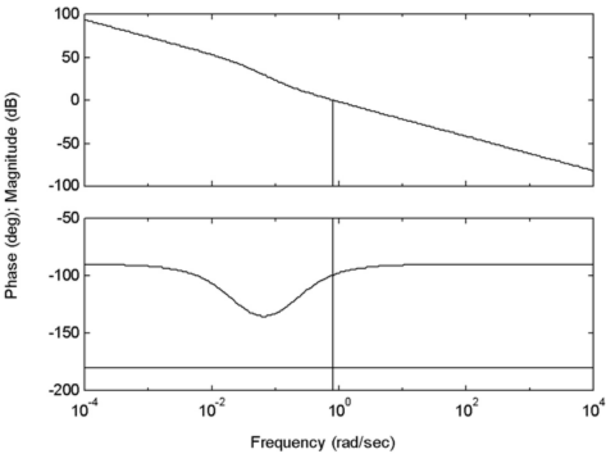


Figure 6: Bode diagram showing magnitude (dB) and phase (deg) for Channel 1 with the compensation provided by the controller transfer function of Equation (27). The gain cross-over frequency is indicated by the vertical line at frequency of about 0.8 rad/s

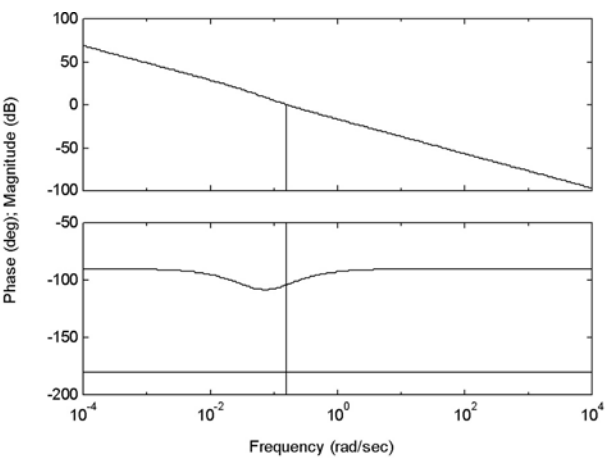


Figure 7: Bode diagram showing magnitude (dB) and phase (deg) for Channel 2 with the compensation provided by the controller transfer function of Equation (28). The gain cross-over frequency is indicated by the vertical line at frequency of about 0.2 rad/s

Figures 6 and 7 show the open-loop Bode plots for Channels 1 and 2, respectively, for these optimised controllers. From these Bode plots it may be seen that the gain crossover frequencies for Channel 1 and Channel 2 are approximately 0.8 rad/s and 0.2 rad/s, respectively. The corresponding phase margins are more than the required value of 70 degrees, in both cases. From the gain-crossover frequencies it is clear that the speed of response for Channel 2 is likely to be about four times slower than for Channel 1, which is consistent with the specifications.

Discrete equivalents of these continuous controllers have been found and an ICAD-based control system has been implemented with a digital controller using a general-purpose computer equipped with analogue-to-digital and digital-to-analogue converters. However, all experimental results presented in this paper are for the continuous control case where the controllers have been implemented using a small general-purpose electronic analogue computer equipped with comparators and switches that can provide limiting integrator action, if required, to avoid integrator saturation.

For purposes of comparison, proportional plus integral controllers have also been designed empirically using the Ziegler-Nichols reaction curve method (see, e.g. [13]). This well-known approach to controller design is based on measurements obtained from simple open-loop tests on the plant. Application of this approach gave the following controller transfer functions:

$$k_1(s) = 6.98\frac{(1+3.96s)}{s} \quad (29)$$

$$k_2(s) = 8.9\frac{(1+3.3s)}{s} \quad (30)$$

It should be noted that the two controller transfer functions found from the application of the Ziegler-Nichols approach (Equations (29) and (30)) are very much closer in terms of parameter values than the two controllers found using the ICAD approach, as given in Equations (27) and (28). This is because of the requirement within the ICAD methodology that the bandwidth values for the two channels should be significantly different.

5 Results

Extensive analysis and simulation studies have been performed using Matlab® and Simulink® to investigate the performance of the system, especially in terms of interactions between the two channels and overall robustness of the control systems.

In the case of the control systems derived using the ICAD approach the performance of the controllers has also been the subject of detailed experimental investigation in the laboratory using the coupled-tanks system hardware. Interactions between the two channels have been investigated both by experiment and through simulation.

For the simulation studies the full nonlinear model has been used, with parameter values as given in Section 2.

5.1 Simulation results

Figure 8 shows typical simulation results for a test in which simultaneous step changes are applied to the reference inputs determining the required levels in the two tanks, using the continuous controllers of Equations (27) and (28). The resulting simulated changes in liquid levels in Tanks 1 and 2 are shown by the upper and lower traces respectively. In the case of Channel 1 the step change of reference imposed is from 199 mm to 228 mm, while for Channel 2 the change is from 165 mm to 198 mm. This test involves input flow values for Tank 1 and Tank 2 which both reach their upper limits for this magnitude of demanded level change.

It can be seen from these simulation results that, although the operating point considered is significantly different from the design point, the design requirements have been satisfied and also that the response of Tank 2 is slower than that of Tank 1, as expected.

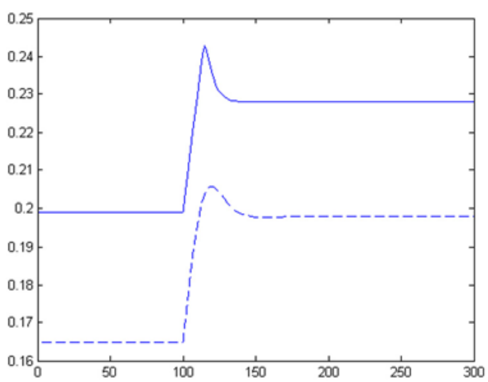
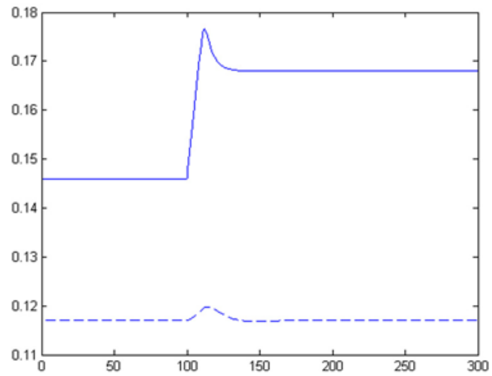


Figure 8: Simulation results found using the nonlinear model with the controllers designed using the ICAD approach for a test in which simultaneous step changes in required levels for Tank 1 and Tank 2 are applied at time $t = 100$ s. The vertical axis represents liquid level (m) while the horizontal axis represents time (s). The level in Tank 1 is represented by the continuous line while the dashed line represents the level in Tank 2

Investigation, through simulation, of interactions between the two channels have involved introducing a step change of the desired level in one channel while maintaining the original set level in the other.

The upper set of simulation results presented in Figure 9a shows the level of liquid in Tank 1 following the application of a step change of reference for Channel 1 at time $t = 100$ s, together with the record for the level in Tank 2. In Figure 9b the lower plot shows the liquid level in Tank 2 following the application, at time $t = 100$ s, of a step change of reference for Channel 2 while the upper trace shows the corresponding level in Tank 1.

a)



b)

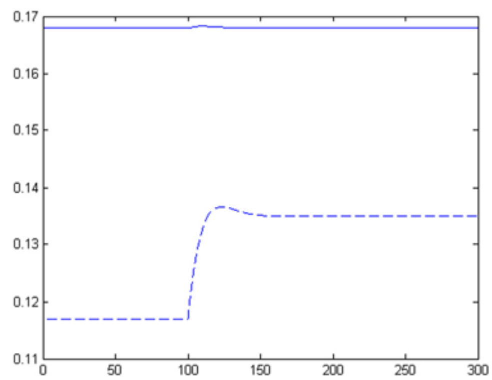


Figure 9: a) Simulated responses of levels (m) in Tank 1 (continuous line) and in Tank 2 (dashed line) versus time (s) when the reference level for Channel 1 is changed. The horizontal axis represents time (s). This test involved use of the nonlinear model with controllers designed using the ICAD approach
b) Simulated responses of levels (m) in Tank 1 (continuous line) and in Tank 2 (dashed line) when the reference level for Channel 2 is changed. This test involved use of the nonlinear model with controllers designed using the ICAD approach. The horizontal axis represents time (s)

These results show that a transient disturbance occurs in the level of Tank 2 when the set level of Channel 1 is changed but that a negligible transient is found in the level of Tank 1 when the set level of Channel 2 is altered by a similar amount. This difference is due to the different bandwidths in the two channels.

Results of an additional simulation test are shown in Figure 10. This involves the simultaneous application of negative step changes of reference for both channels at time $t = 100$ s. The demanded changes lead, transiently, to a complete cut-off of input flow for both tanks for a period of about 20 s at the time when the reference values are changed, as can be seen from the almost straight-line form of the negative-going responses in that part of the record.

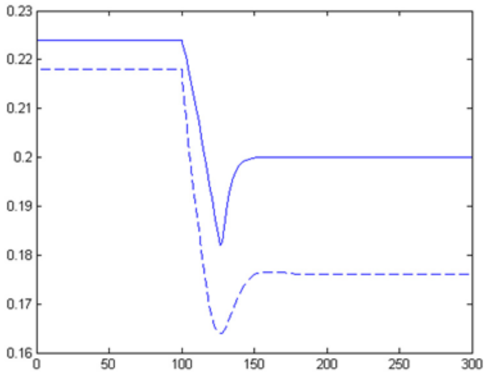


Figure 10: Simulation results showing liquid levels (m) for Tank 1 (continuous line) and Tank 2 (dashed line) for large negative step changes of reference. The horizontal axis represents time (s)

The closed-loop performance of the system with the proportional plus integral controllers designed using the Ziegler-Nichols reaction curve approach (see, e.g. [13]) was investigated through simulation. It was found that for small changes of the reference inputs the two-input two-output system with these controllers behaved very much in accordance with expectations (as shown in Figure 11).

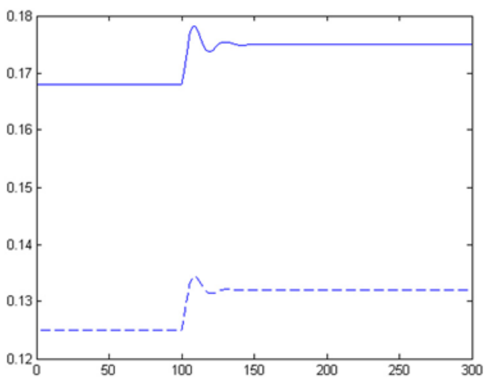


Figure 11: Simulation results found using the nonlinear model with controllers designed using the Ziegler-Nichols approach for a test involving relatively small changes of reference level. The vertical axis represents liquid level (m) while the horizontal axis represents time (s). The level in Tank 1 is represented by the continuous line while the dashed line represents the level in Tank 2

For large positive changes of reference (similar to those applied in obtaining the results shown in Figure 8 for the ICAD design) the responses for the control system designed using the Ziegler-Nichols approach are found to be much more oscillatory, as shown in Figure 12. This is also the case for the transients found for large negative reference changes, as shown in Figure 13.

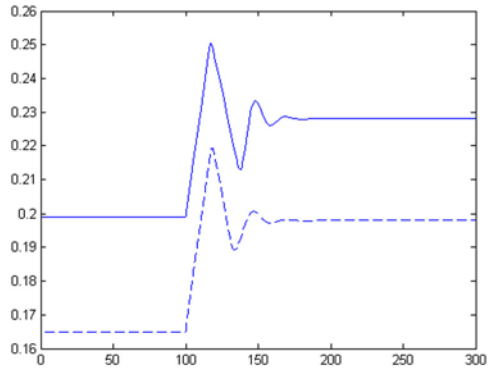


Figure 12: Simulation results found using the nonlinear model with controllers designed using the Ziegler-Nichols approach for a test similar to that of Figure 8. The vertical axis represents liquid level (m) while the horizontal axis represents time (s). The level in Tank 1 is represented by the continuous line while the dashed line represents the level in Tank 2

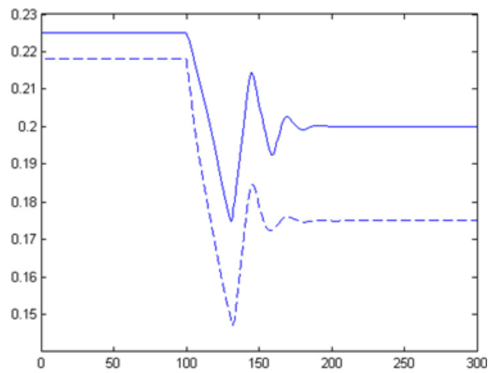


Figure 13: Simulation results for the nonlinear model with controllers designed using the Ziegler-Nichols approach for a test involving large negative step changes of reference. The vertical axis represents liquid level (m) while the horizontal axis represents time (s). The level in Tank 1 is represented by the continuous line, whereas the dashed line is the level in Tank 2

Responses found for simulated situations involving interactions between the two tanks were also more oscillatory in nature and varied more with operating point than those found using the controllers designed using the ICAD approach.

5.2 Experimental results

As implemented using operational amplifiers and the associated passive components, the two controllers corresponded to the transfer functions (as given in Equations (27) and (28)) that resulted from the final optimisation stage of the design process. These are, of course, also the controller transfer functions used in the simulation studies discussed in Section 5.1.

Experimental results for the control system, when implemented with these controllers, are shown in Figure 14 for the case involving two simultaneous changes of reference. The results are almost identical in terms of steady state performance to the simulated results of Figure 8 for the same test conditions, and are very similar in terms of the settling time of the transients. As in the simulation results, the inputs both reach their limits during the transients.

The main difference observed between the experimental results of Figure 14 and the simulation results of Figure 8 is that the transients found experimentally (especially for the level in Tank 1) are more oscillatory than those found through simulation. Similar findings have been obtained for equivalent tests at other operating points, and this suggests strongly that there are imperfections within the model of the two-tank system. Exactly what the modelling errors might be is not, of course, clear from the information from these closed-loop system tests alone.

Although the results shown in Figures 8 and 14 are for one specific operating condition, comparison of experimental and simulation results for a range of different conditions has shown good overall agreement.

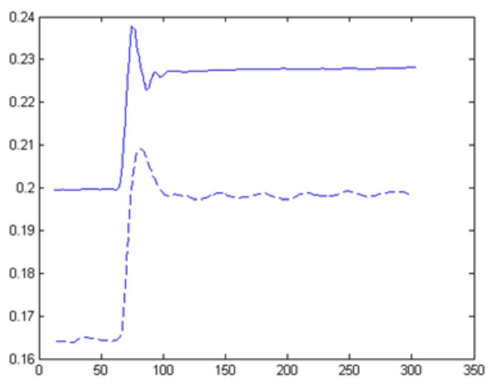
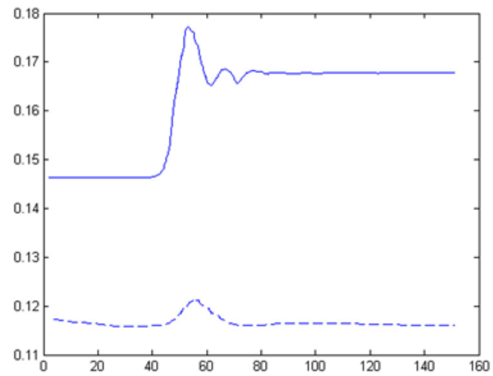


Figure 14: Experimental results for conditions equivalent to those of the simulation results of Figure 8. The continuous line shows the measured liquid level (m) in Tank 1 while the dashed line shows the measured level in Tank 2. The horizontal axis represents time (s)

The experimental investigation of interactions between channels produced results shown in Figures 15a and 15b, which can be seen to correspond closely to the corresponding simulation results shown in Figures 9a and 9b.

Experimental results for a test involving simultaneous large negative changes of reference value for both channels simultaneously are shown in Figure 16. These results are very similar in character to the simulated results of Figure 10. As in the simulation, the controlled flows for Tank 1 and Tank 2 reach limiting values (zero) during the transient period.

a)



b)

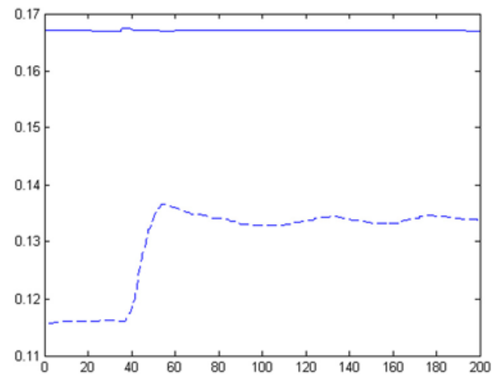


Figure 15: a) Experimental results, equivalent to the simulation results of Figure 9a, involving application of a step change of reference for Channel 1 while the reference input for Channel 2 is held constant. The record for liquid depth (m) in Tank 1 is shown by a continuous line and for Tank 2 by the dashed line. The horizontal axis represents time (s)
b) Experimental results, equivalent to the simulation results of Figure 9, involving application of a step change of reference for Channel 2 while the reference for Channel 1 is held constant. The record for liquid depth (m) in Tank 1 is shown by a continuous line and for Tank 2 by the dashed line. The horizontal axis represents time (s)

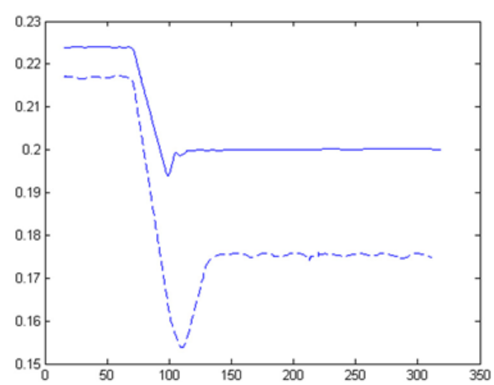


Figure 16: Experimental results showing liquid levels (m) for Tank 1 (continuous line) and Tank 2 (dashed line) for large negative step changes of reference. The horizontal axis represents time (s)

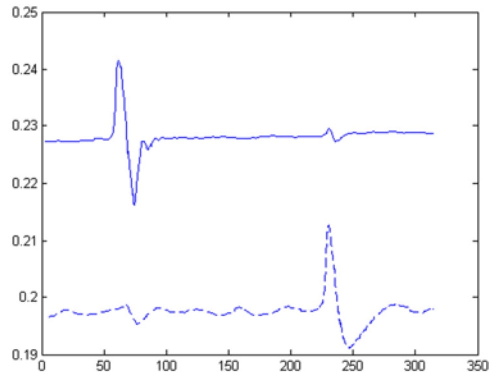
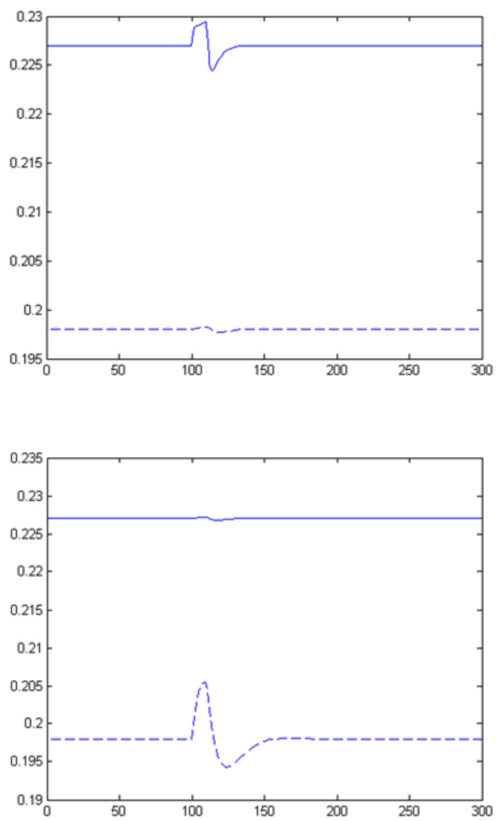


Figure 17: Experimental results for a test involving the addition of a small volume of water to Tank 1 (continuous trace) and to Tank 2 (dashed line) in turn. The horizontal axis represents time (s)



a)

b)

Figure 18: a) Results of a simulated test involving the addition of a small volume of water to Tank 1 (continuous line). The level in Tank 2 is shown by the dashed line. The horizontal axis represents time (s) b) Results of a simulated test involving the addition of a small volume of water to Tank 2 (dashed line). The level in Tank 1 is shown by the continuous line. The horizontal axis represents time (s)

The behavior of the control system when subjected to external disturbances is also of great practical importance. Figure 17 shows some typical experimental results where external disturbances have been introduced by adding, in as short a period of time as possible, a disturbance in the form of a measured volume of water to each of the tanks in turn, with feedback control loops applied. The upper plot shows the level in Tank 1 for a reference input of 227 mm, while the lower plot shows the level in Tank 2 for a reference input of 198 mm. The disturbance inputs are applied by the rapid addition of water, from a beaker, to Tank 1 and addition of a similar volume to Tank 2 at about time $t = 225$ s. The effects of each of these disturbances on each channel are clearly visible in these records.

The results show the distinctive actions of the two channels in countering the effects of the disturbance inputs. The levels for both channels return to their set values after the disturbances, with transients of acceptable magnitude and duration. As with the tests involving changes of reference, it is clear (as would be expected) that the speed of response to disturbances is influenced by the choice of bandwidths for the two channels.

Similar results have been obtained through simulation, but quantitative comparisons are difficult for this type of test because it is hard to reproduce the detailed time-course of the disturbance input within the simulation. Figures 18a and 18b show typical simulation results for disturbance tests which are approximately equivalent to the experiments of Figure 17. The experimental and simulation results are seen to be qualitatively consistent.

5.3 Results of additional simulation-based investigations

One interesting practical finding, which has been fully supported by simulation results, is that control of the level in Tank 1 can only be achieved for conditions in which the demanded level in Tank 1 is equal to or greater than that in Tank 2. This is understandable in terms of the physics of the system since Tank 1 has only one outlet (to the second tank), whereas Tank 2 has two outlets (one to the first tank and the second through the drain pipe). If the demanded value of H_2 is greater than the demanded level of H_1, liquid will flow into Tank 1 from Tank 2 as well as from the input but no liquid will flow out. Since the input flow cannot become negative, satisfactory control of the level in Tank 1 is impossible in these conditions. Typical experimental results are shown in Figure 19 and these demonstrate, in this specific case, that a demanded level of 0.105 m in Tank 1 cannot be achieved in combination with a larger demanded level (in this case 0.131 m) in Tank 2. What happens in practice is that the final levels in both tanks become equal to the final demanded level in Tank 2. Figure 20 shows results obtained from simulation for a very similar set of conditions. Simulation investigations have confirmed that the addition of a drain pipe to the first tank eliminates this problem and would allow independent control of the two levels for any combination of reference values.

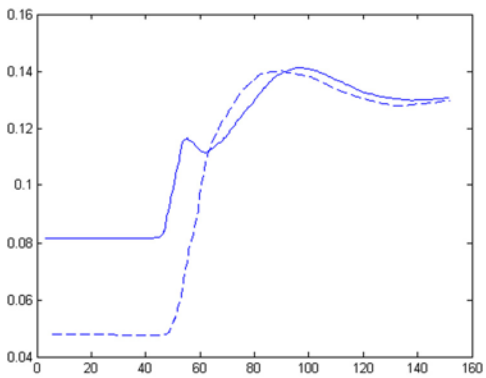


Figure 19: Levels (m) found in Tank 1 (continuous line) and Tank 2 (dashed line) for a demanded change of the reference for Channel 1 from 0.082 m to 0.105 m and for Channel 2 from 0.048 m to 0.131 m. The horizontal axis represents time (s)

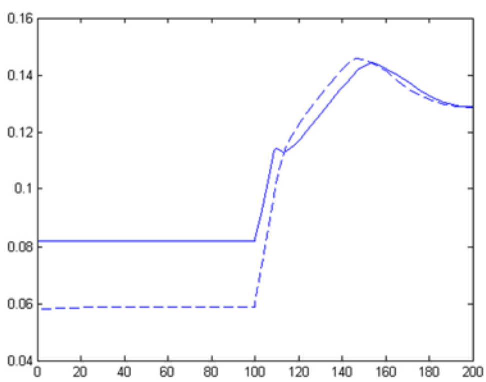


Figure 20: Simulated results for a test which involves conditions which are very similar to those of the experiment of Figure 16. In this case, following the step changes of reference inputs, the demanded level in Tank 2 is again greater (0.131 m) than the demanded level in Tank 1 (105 mm). Here the continuous line again represents the liquid level in Tank 1 and the dashed line the level in Tank 2. The horizontal axis represents time (s)

Another area for further investigation through simulation relates to tests of robustness to changes within the plant. These have involved, for example, the introduction of sudden changes of the cross-sectional area of the outlet drain orifice, or of the cross-sectional area of the orifice responsible for the coupling between Tank 1 and Tank 2.

Experimental testing is straightforward in the case of the outlet from Tank 2, for which the drain tap can be used, but investigation of changes of the inter-tank orifice area presents practical difficulties since the variation of cross-sectional area normally requires the removal or insertion of a rubber bung for one of the three orifices in the partition that separates the two tanks. Even partial closing and re-opening of the outlet drain tap is difficult to achieve manually in a precise and repeatable fashion. Simulation can therefore be used to advantage to investigate the performance of the system for this type of change.

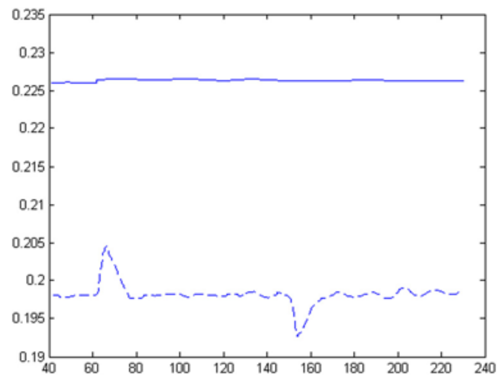


Figure 21: Results of an experiment involving partial closure and re-opening of the drain tap from Tank 2. The action of closure occurs at about $t = 60$ s and the re-opening takes place at about $t = 150$ s. The continuous line shows the level (m) in Tank 1 and the dashed line represents the level (m) in Tank 2. The horizontal axis represents time (s)

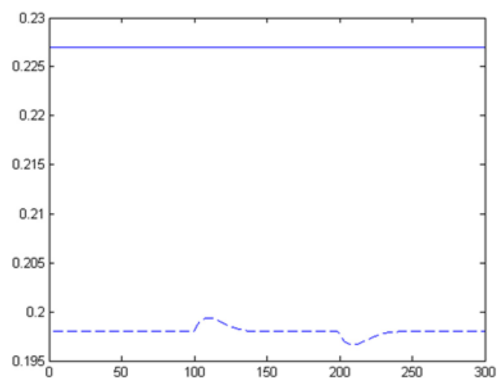


Figure 22: Results from a simulation involving instantaneous changes of the cross-sectional area of the orifice representing the drain tap from Tank 2. Partial closure occurs at $t = 100$ s and the re-opening takes place at $t = 300$ s. The continuous line shows the level (m) in Tank 1 and the dashed line represents the level (m) in Tank 2

Inevitably, the results of simulation tests differ slightly from tests carried out on the real system. Typical experimental results showing the effects of changes of drain tap opening and changing the cross-sectional area of the inter-tank orifice are given in Figure 21. Results from simulation, for instantaneous changes of the cross-sectional area of the orifice representing the drain-tap and outlet pipe, are shown in Figure 22 and these are broadly similar to the experimental findings of Figure 21. Both the simulation results and the experimental findings confirm that the robustness properties of the two-input two-output control system, as displayed in this test, are satisfactory. Transients for Tank 2 are significantly larger than for Tank 1 as could be expected from the bandwidths of the two channels.

6 Discussion and conclusions

The work reported in this paper illustrates the use of the ICAD analysis and design approach for a practical application that involves significant nonlinearities in terms both of control input limits and inherent nonlinearity of the plant model. Analysis of the two-input two-output system within the ICAD framework provides helpful insight which can be used in the design and implementation of the control system.

Comparisons between simulation and experimental results also provide useful information about the system performance and about limitations of the plant model. A previous journal paper [9] reporting the application of the ICAD methodology to the same system was concerned with issues of controller parameter tuning and did not consider the response to disturbances or address robustness issues.

It can be concluded that the coupled tanks equipment provides a useful test-bed for investigation of issues of nonlinear system modeling, multivariable control system design and implementation. The availability of a comprehensive nonlinear model of the system, together with linearised representations appropriate for control system design, also makes this system suitable for the teaching of practical aspects of multi-input multi-output control system analysis and design using ICAD or other approaches. It is believed that the work reported in this paper could provide the basis for a useful case-study (most probably for use at postgraduate level) on the ICAD methodology. This could also illustrate the benefits of bringing together simulation and experimental testing within the processes of control system design and implementation.

Differences between simulation results and experimental results are believed to relate mainly to limitations in the representation of the plant and especially the relationship used to describe the output flow from the second tank within the nonlinear model. This aspect of the coupled-tanks system model has been discussed in previous model validation studies for this system (e.g. [7, 10, 11]) and is the subject of ongoing investigations.

Simulation results show that broadly satisfactorily results can also be obtained for this plant with proportional plus integral controllers designed empirically using the Ziegler-Nichols reaction curve method. However, results found using that approach have given responses, for the types of tests described in this paper, that tend to be more oscillatory than those found using the ICAD methodology, and indicate some issues of closed-loop system robustness to changes of operating point, variation of the magnitude of reference changes and the magnitude of disturbances.

Claims that the ICAD approach can provide enhanced performance compared with other available design methods would be inappropriate on the basis of the limited results presented in this single application. However, it is believed that the ICAD methodology brings more physical insight to the design process for multi-input multi-output systems, and it must also be remembered that this approach is not restricted to one single form of controller.

Acknowledgement

This paper is a modified and extended version of a paper [8] presented at EUROSIM 2010 (the 7th EUROSIM Congress on Modelling and Simulation), which was organized by staff of the Department of Computer Science and Engineering, Czech Technical University in Prague (CTU). The Congress took place in Prague during the period 6-10 September, 2010.

The author must thank Jin Lin Chiang who, during undergraduate project work at the University of Glasgow [12], carried out experiments from which some of the results in this paper have been derived.

The author also wishes to thank Professor John O'Reilly of the University of Glasgow for many valuable discussions about ICAD methods.

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